An effective method of solving the covariance equation for statistical modeling

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Abstract — **The backward propagation of variance (BPV) technique for statistical modeling has proven to be efficient for calculating statistical variations of model parameters including some correlations coefficients. In this paper the complete covariance equation is set up, normalized and solved by a least square method. The target PCM parameter deviations are derived from the spec limits of the process control monitoring.**

Keywords-component; Backward propagation of variance; Device correlation; Statistical modeling; Covariance

I. INTRODUCTION

The consideration of process deviations of semiconductor devices in integrated circuits is very important in order to get a forecast of the yield that the circuit will have after being processed. This is commonly done during the design process with Monte Carlo circuit simulations. The forecast of the circuit yield is the more exact the more precise the statistical variations of the underlying model parameters are. To choose the right set of model parameters and to let them vary with the correct values is a difficult task. The data bases for deriving process variations are typically very few.

II. PCM AS DATA BASE FOR STATISTICAL MODELING

Often the only available statistical measurement data are those that are used for controlling the process in the semiconductor fab. On narrow test structures placed between the productive dies device parameters can be found that are useful for statistical modeling among many other parameters that are not related to devices but important for controlling the process. Typical parameters for MOS transistors are e.g. the threshold voltage in the linear and saturation region (VTL and VTS), the saturation current (ISAT) and the Early voltage. The small set of MOS transistor geometries that can be found contains usually the minimum values of width and length and bigger ones that are assumed to be free from narrow or short channel effects. Seldom also capacitors are available on PCM structures. The data coming from PCM measurement are incomplete from the view of the modeling engineer, because mostly not all the device types of a technology are present in PCM. Additionally the device sizes are restricted due to the limited space between the dies (usually several tens of microns). Furthermore temperatures other than room temperatures are not taken into account.

In the procedure of controlling the manufacturing process the spec limits (lower and upper spec limit LSL and USL) and the target values for each PCM parameter are the most important assessment criteria. Based on the limits, the mean value μ and the standard deviation σ the process capability index (CPK) is calculated for each parameter. The CPK value describes the quality of the process regarding process deviations.

$$CPK = \min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right) \tag{1}$$

In statistical process control normally a certain minimum CPK value that may be individual or common for PCM parameters is guaranteed by the fab (CPK_{min}). CPK values for key PCM parameters are determined and reviewed based on the data that are measured during a certain period of time (e.g. one month). If the CPK value of a parameter comes critically near CPK_{min} measures have to be started that drive the CPK value back to higher values.

III. PCM SIMULATION

A possible vehicle for checking the model-hardware correlation between the results of statistical circuit simulation and PCM is to simulate a test bench which outputs represent PCM parameters. For this purpose it is necessary to reproduce exactly the operating conditions being applied by the PCM tester as well as the calculations done by its measurement routine. The first obvious step is to check the results of a nominal test bench simulation against the PCM targets. The second step is to set up a Monte Carlo simulation with the test bench and compare the standard deviations of the simulated output parameters with the predetermined desired values of PCM standard deviations.

However, it is not clear what the PCM standard deviations that the method is targeting on should be. Trying to reproduce exactly σ of the measurement data seems not to be the best way, especially if the available measurement samples are observed during a limited period of time. A distribution for this period may not stay constant in the future. It is allowed to shift or to broaden unless the CPK value stays above CPK_{min}. The challenge of a modeling engineer is to make a distribution forecast of every PCM parameter for the whole lifetime of the technology. One possibility to do so is to multiply σ of the measured PCM parameters with a factor of safety S_{factor}.

$$\sigma_{factor} = S_{factor}\sigma_{meas} \tag{2}$$

The individual values of σ_{factor} are physically consistent as they are derived from real measurement data. However they might not meet the guarantee criteria of the fab.

Another possibility is to use the spec limits of PCM parameters and their guaranteed minimum value CPK$_{min}$. With

$$\sigma_{CPK} = \frac{USL - LSL}{6\sigma\, CPK_{min}} \tag{3}$$

the target deviation is fixed to a value that corresponds to a process capability value of CPK$_{min}$. The determination of σ_{CPK} does not need any measurement results. Statistical models that correlate to σ_{CPK} guarantee the same qualities of process stability as the process controlling in the fab itself. This method however presumes that the spec limits are defined roughly in a consistent way. The CPK values of measured data for a time period have to be approximately all on the same level. In this paper we follow this approach for defining the targets of PCM simulation for a 350nm BCD process for a general CPK$_{min}$ value of 1.5.

IV. THE COVARIANCE MATRIX EQUATION

Various methods for the determination of SPICE model parameter variations in order to describe process deviations have been proposed [2]. In [1] a method called *backward propagation of variance technique* was extended by the determination of some correlation coefficients. We follow this approach and extend it to an effective procedure for calculating the complete set of model parameter variations and correlation coefficients with simple matrix algebra.

The relationship between the mean free deviations of one set of PCM parameters $\vec{p}$ and the mean free variations of a model parameters set $\vec{m}$ is assumed to be linear.

$$\vec{p} = S \cdot \vec{m} \tag{1}$$

S is the sensitivity matrix of the simulated PCM parameters deviations with respect to the model parameters variations. Combining the parameter vectors of many instances, we get the matrix form

$$P = S \cdot M\,, \tag{2}$$

with n simulation samples of model parameters

$$M = (\vec{m}_1, \ldots, \vec{m}_n) \tag{3}$$

and m PCM measurement values

$$P = (\vec{p}_1, \ldots, \vec{p}_m)\,. \tag{4}$$

Therefore, with elementary matrix operations the covariance of the PCM parameters can be written as [3]

$$\operatorname{Cov}(P) = \operatorname{Cov}(S \cdot M) = S \cdot \operatorname{Cov}(M) \cdot S^T \tag{5}$$

Eq. (5) represents a linear model of the relations between PCM and model parameter covariances. Due to the fact that PCM parameters (as well as model parameters) have different orders of magnitude a normalization step is necessary in order to weight the PCM parameters equally. This leads to the normalized sensitivity matrix.

$$S_{norm} = \begin{bmatrix} \frac{1}{\sigma_1^P} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\sigma_n^P} \end{bmatrix} \cdot S \tag{6}$$

The target PCM deviations are denoted σ_i^P . In the normalized sensitivity matrix all rows have approximately the same weights. In the normalized form the covariance equations can be written as

$$S_{norm} \cdot \operatorname{Cov}(M) \cdot S_{norm}^T = \operatorname{Corr}(P) \tag{7}$$

The matrix Corr(P) is extracted from PCM database with correlation analysis that is available in standard database software. S$_{norm}$ is got from a sensitivity analysis of the test bench. To simplify the problem, Corr(P) can be written in a symmetric form which is denoted by GG^T . This can be achieved by matrix diagonlization.

$$\begin{aligned} \operatorname{Corr}(P) &= Q^T \cdot \lambda \cdot Q \\ G &:= Q^T \cdot \sqrt{\lambda} \\ GG^T &= \operatorname{Corr}(P) \end{aligned} \tag{8}$$

Eq. (7) can be solved, if a matrix H can be found that fulfils

$$S_{norm} \cdot H = G \tag{9}$$

The matrix HH^T is identified with the covariance matrix Cov(M). Eq. (9) has a unique solution if there are as many model parameters as PCM parameters for each device. If there are more PCM parameters it is solved by a least square fit, see e.g. in [4].

$$\operatorname{Corr}(M) = \left(\frac{\operatorname{Cov}(M_i, M_j)}{\sigma_i^M \sigma_j^M} \right)_{i=1..n,\, j=1..n} \tag{10}$$

According to Eq. (11) the correlation coefficients are calculated. From the diagonal elements of Cov(M) the standard deviations of the model parameters are calculated by

$$\sigma(m_i) = \sqrt{\operatorname{Cov}(\vec{m}_i, \vec{m}_i)} \tag{11}$$

Both are used directly in the parameter correlation section of the model parameter set.

V. Results

As an example two PMOS transistor types are considered namely a low-voltage and a high-voltage MOSFET (MP and MPH). These two devices are technically related because they have the same gate oxide thickness and also see some identical process steps. There are nine PCM parameters measured for these two device types. For MP there are MP_L_VTL, MP_L_ISAT, MP_S_ISAT and MP_S_RON and for MPH the parameters MPH_VTL, MPH_VTS, MPH_ISAT and MPH_RON are existing. There is also a common parameter T_GOX which is the oxide thickness calculated from a measured MOS capacitance. An “S” at the end of a parameter name nominates a short and an “L” a long channel device. They are denoted as GLOBAL_TOX_PRC, GLOBAL_LINT_PRC, MP_VTH0_PRC, MP_U0_PRC, MP_VSAT_PRC, MPH_VTH0_PRC, MPH_U0_PRC and MPH_RDSON_PRC. Here we find variations for model parameters of BSIM3v3 parameters VTH0, U0, TOX, LINT and VSAT. The parameter RDSON is a factor that is used in the sub-circuit of the HVPMOS model for the modeling of the on-resistance. Gate oxide thickness TOX and channel length offset LINT as global parameters have an impact on both device types.

After determining the sensitivity matrix the normalized covariance equation is solved with respect to model parameter variations and correlation coefficients (see TABLE I. and 0).

TABLE I. Model Parameters Variations

Model Parameter	Unit	values
GLOBAL_TOX_PRC	%	1,843E-02
GLOBAL_LINT_PRC	m	1,404E-08
MP_VTH0_PRC	V	1,783E-02
MP_U0_PRC	%	1,697E+00
MP_VSAT_PRC	%	6,226E+03
MPH_VTH0_PRC	V	1,905E-02
MPH_U0_PRC	%	4,922E+00
MPH_RDSON_PRC	%	1,922E-01

TABLE II. Model Parameters Correlation Coefficients

Model Parameters	GLOBAL_TOX_PRC	GLOBAL_LINT_PRC	MP_VTH0_PRC	MP_U0_PRC	MP_VSAT_PRC	MPH_VTH0_PRC	MPH_U0_PRC	MPH_RDSON_PRC
GLOBAL_TOX_PRC	1,00	0,27	-0,37	0,48	0,27	-0,37	0,04	0,08
GLOBAL_LINT_PRC	0,27	1,00	-0,48	-0,49	0,50	-0,43	0,34	0,57
MP_VTH0_PRC	-0,37	-0,48	1,00	0,01	-0,54	0,93	0,09	-0,02
MP_U0_PRC	0,48	-0,49	0,01	1,00	0,01	-0,04	0,07	-0,45
MP_VSAT_PRC	0,27	0,50	-0,54	0,01	1,00	-0,49	0,23	0,33
MPH_VTH0_PRC	-0,37	-0,43	0,93	-0,04	-0,49	1,00	0,09	0,02
MPH_U0_PRC	0,04	0,34	0,09	0,07	0,23	0,09	1,00	0,21
MPH_RDSON_PRC	0,08	0,57	-0,02	-0,45	0,33	0,02	0,21	1,00

Remarkable is the fact that model parameters that belong to the same device have noticeable high values. The functional relations between model parameters on the one hand and PCM parameters on the other that are given by model equations (e.g. in BSIM3v3) do not seem to be sufficient.

Model parameter variations and correlation coefficients are used for a Monte Carlo simulation with 300 runs. From the output samples of this simulation the simulated PCM deviations are determined. The following TABLE III. shows the standard deviations of simulated PCM parameters in comparison to the target values. All simulated parameter deviations differ less than 10% from the target values.

TABLE III. Solution for PCM Standard Deviations

PCM Parameter	Target	Simulation	Deviation
T_GOX	4,444E-01	4,307E-01	-3,09%
MP_L_VTL	1,867E-02	1,735E-02	-7,04%
MP_L_ISAT	1,544E-06	1,500E-06	-2,90%
MP_S_ISAT	1,778E-05	1,827E-05	2,75%
MP_S_RON	3,556E+01	3,630E+01	2,09%
MPH_VTL	2,333E-02	2,127E-02	-8,84%
MPH_VTS	2,000E-02	2,119E-02	5,95%
MPH_ISAT	6,667E-06	6,778E-06	1,67%
MPH_RON	2,000E+02	2,124E+02	6,20%

The simulated PCM correlation matrix is shown in TABLE IV. and the absolute differences to the target correlation values in TABLE V.

TABLE IV. SIMULATED PCM CORRELATIONS

PCM Parameters	T_GOX	MP_L_VTL	MP_L_ISAT	MP_S_ISAT	MP_S_RON	MPH_VTL	MPH_VTS	MPH_ISAT	MPH_RON
T_GOX	1,00	-0,42	0,55	0,07	-0,02	-0,41	-0,43	0,51	0,17
MP_L_VTL	-0,42	1,00	-0,88	0,00	0,23	0,91	0,92	-0,66	-0,15
MP_L_ISAT	0,55	-0,88	1,00	-0,10	-0,28	-0,80	-0,83	0,60	0,33
MP_S_ISAT	0,07	0,00	-0,10	1,00	0,90	-0,06	0,01	0,52	-0,41
MP_S_RON	-0,02	0,23	-0,28	0,90	1,00	0,15	0,21	0,35	-0,39
MPH_VTL	-0,41	0,91	-0,80	-0,06	0,15	1,00	0,98	-0,73	-0,05
MPH_VTS	-0,43	0,92	-0,83	0,01	0,21	0,98	1,00	-0,70	-0,14
MPH_ISAT	0,51	-0,66	0,60	0,52	0,35	-0,73	-0,70	1,00	0,03
MPH_RON	0,17	-0,15	0,33	-0,41	-0,39	-0,05	-0,14	0,03	1,00

TABLE V. CORRELATION COEFFICIENTS, DIFFERENCES BETWEEN SIMULATION AND MEASUREMENTS

PCM Parameters	T_GOX	MP_L_VTL	MP_L_ISAT	MP_S_ISAT	MP_S_RON	MPH_VTL	MPH_VTS	MPH_ISAT	MPH_RON
T_GOX	0,00	-0,03	0,04	0,01	0,02	-0,03	-0,04	0,08	0,00
MP_L_VTL	-0,03	0,00	0,03	0,05	0,01	-0,01	-0,02	-0,02	-0,07
MP_L_ISAT	0,04	0,03	0,00	-0,05	0,01	0,03	0,03	0,00	0,04
MP_S_ISAT	0,01	0,05	-0,05	0,00	-0,05	0,03	0,01	-0,07	-0,03
MP_S_RON	0,02	0,01	0,01	-0,05	0,00	-0,02	-0,04	-0,06	-0,04
MPH_VTL	-0,03	-0,01	0,03	0,03	-0,02	0,00	0,00	-0,03	-0,08
MPH_VTS	-0,04	-0,02	0,03	0,01	-0,04	0,00	0,00	-0,04	-0,03
MPH_ISAT	0,08	-0,02	0,00	-0,07	-0,06	-0,03	-0,04	0,00	0,03
MPH_RON	0,00	-0,07	0,04	-0,03	-0,04	-0,08	-0,03	0,03	0,00

VI. CONCLUSIONS

In this paper we propose a new methodology of solving the covariance equation for statistical modeling. Target PCM parameter deviations are defined according to process assessment criteria, while the target correlation coefficients are derived directly from PCM measurement data. Our approach with simple analytic matrix algebra appears to be straight forward. The results show good agreement between simulated and measured PCM correlations and standard deviations.

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